Cooling Torsional Nanomechanical Vibration by Spin-Orbit Interactions

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We propose and study a spin-orbit interaction based mechanism to actively cool down the torsional vibration of a nanomechanical resonator made by semiconductor materials. We show that the spin-orbit interactions of electrons can induce a coherent coupling between the electron spins and the torsional modes of nanomechanical vibration. This coherent coupling leads to an active cooling for the torsional modes via the dynamical thermalization of the resonator and the spin ensemble.

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Introduction.-Fast developments of nano-fabrication technology enable us to manufacture the nanomechanical resonators (NAMRs) with high frequency and high quality[1]. These artificial systems have attracted more and more attentions for their application potentials, such as ultra-sensitive probe for tiny displacement[2], single spin detector[3], and physical implementations of quantum information[4]. On the other hand, it is believed that a high frequency NAMR (e.g. with GHz oscillation) will exhibit various quantum effects in this macroscopic system at low temperatures[1]. Therefore, the studies on NAMR are relevant to fundamental problems like quantum measurement[5].

To give prominence to the quantization effects of NAMR, the crucial issue depends on whether we can cool it down to the vibrational ground state. Actually, interacting with the surrounding environment, the NAMR reaches a thermal equilibrium through the relaxation. The mean thermal occupation number of the NAMR, which is determined by the environment temperature and the vibration frequency, is always larger than unity for most of the present nanomechanical systems even at the sub-Kelvin temperature. Accordingly, besides decreasing the environment temperature, special cooling mechanism is needed to be invented to further reduce the thermal occupation number.

Recent experiments have shown that the micromechanical systems can be cooled down by radiation pressure[6] or by coupling to a single electron transistor[7]. And several theoretical scenarios were proposed based on different cooling mechanism, such as laser excitation of phonon sideband of a quantum dot[8], and periodic coupling to a Cooper pair box[9], et al. All the cooling mechanisms mentioned above are only focused on the flexural mechanical modes. In this Letter, we propose an active cooling mechanism for the quantized torsional nanomechanical modes (TNMMs)[10–12] of the NAMR.

Similar to the displacements and momentums for the flexural modes, the TNMMs are described by the oscillation of the torsion angles and the angular momentums of the NAMR. Thus, it is quite natural to to consider how to couple the TNMMs to the electron spin degrees of freedom, which also behave as angular momentums. Recently, a novel interaction between the spin current in semiconductor materials and the NAMR induced by the spin-orbit coupling is discovered in Ref. [13]. This pioneer investigation is carried out in the semi-classical regime to provide the possibility for detecting spin current by the NAMR[13, 14].

Here, we present a microscopic quantum mechanical description for the coherent coupling between the electron spins and the TNMM. With this coherent coupling, we show that the mean occupation number of the NAMR can be significantly reduced. Thus, the NAMR is actively cooled down and can be brought to the quantum realm.

Model for spin-strain coupling.-We consider a doubly clamped nanomechanical rod with length l, width d, and hight h (Fig.1). Similar to the structure considered in Ref. [13], the semiconductor layer grows on an insulating substrate (Fig. 1(b)). The electrons flowing through the nanomechanical rod are confined in the semiconductor layer. For the nth normal mode, the torsion angle

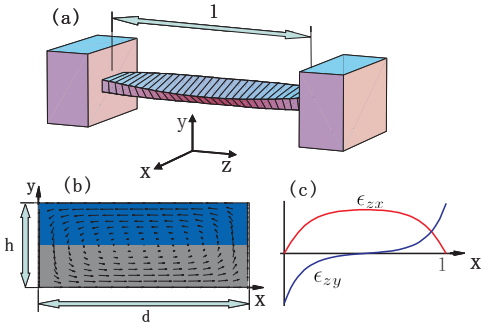

FIG. 1: (a) Schematic illustration of torsional vibration of the nano-mechanical rod. (b) The cross section of the nanomechanical rod and its strain distribution. Electrons flowing through the nanomechanical rod are confined in the semiconductor layer (blue), which grows on an insulating substrate (grey). (c) A typical strain distribution of ϵ_{zx} (red) and ϵ_{zy} (blue). It is shown that ϵ_{zx} is an even function with respect to x, while ϵ_{zy} is odd.

of the NAMR is $\theta_n(z,t) = \theta_{n0}\sin(n\pi z/l)\cos(\omega_n t)$ with frequency ω_n and amplitude θ_{n0}.

According to the isotropic elastic theory[15, 16], the strain tensor field has non-vanishing components $\epsilon_{xz} = \epsilon_{zx} = \theta'(z)(\partial f/\partial y)$ and $\epsilon_{yz} = \epsilon_{zy} = -\theta'(z)(\partial f/\partial x)$ where $\theta'(z) = \partial\theta/\partial z$, and the function $f(x,y)$ is determined by the cross section geometry of the NAMR[15]. We illustrate the strain field explicitly in Fig. 1(b).

To describe the quantum phenomena of the torsional oscillation of the NAMR, we apply the standard canonical quantization procedure by modeling the NAMR as the many-harmonic-oscillator with Hamiltonian $H_{\rm NAMR} = \sum_n \hbar\omega_n(a_n^\dagger a_n + 1/2)$. Here, the boson operator $a_n = I^{1/4}\sqrt{M\omega_n/2\hbar}(\theta_{n0} + i\dot{\theta}_{n0}/\omega_n)$ is introduced through the effective mass $M = \rho\sqrt{I}l/2$, where ρ is the mass density, and $I = \int(x^2+y^2)dA$ is the rotating inertia about z axis for A being the cross section area.

In the following, we will focus on the fundamental ($n = 1$) TNMM at low temperatures and thus omit the subscript index of the boson operators. Then the *quantized* strain field along the NAMR is described by $\epsilon_{zx} = (\partial f/\partial y)F(z), \epsilon_{zy} = -(\partial f/\partial x)F(z)$, where $F(z) = \epsilon_1\cos(qz)(a^\dagger + a)$, $\epsilon_1 = \pi l^{-1}(\hbar/2M\omega_1\sqrt{I})^{1/2}$ and $q = \pi/l$.

As pointed out in Ref. [13], the spins in a nanomechanical rod can be coupled to the strain field in narrow band semiconductor materials by the Hamiltonian[17]

$$H_{\rm SO}^{3D} = \alpha[\sigma_x(\epsilon_{xz}k_z - \epsilon_{xy}k_y) + c.p.], \tag{1}$$

where α is the coupling strength, and $\sigma_{x,y,z}$ and $\hbar k_{x,y,z}$ are the Pauli matrices and the momentum components of the electron respectively. Here, 'c.p.' stands for cyclic permutation of x, y and z. Due to the confinement in the x and y directions, the spacial wave function of the electron can be separated as $\psi(x,y,z) = \varphi(x,y)\exp(ik_z z)$, where $\varphi(x,y)$ is determined by the transverse confinement in the x and y directions. We further assume that this confinement is so strong that we can take an average over $\varphi(x,y)$ to obtain an effective Hamiltonian $H_{\rm SO} = \langle\varphi(x,y)|H_{\rm SO}^{3D}|\varphi(x,y)\rangle$. Considering the parity of the non-vanishing strain components (see Fig. 1(c)), the Hamiltonian $H_{\rm SO}^{3D}$ is then reduced from three-dimension to one-dimension effectively, i.e. $H_{\rm SO} = \alpha\mathcal{E}k_z(a^\dagger + a)\cos(qz)\sigma_x$, where $\mathcal{E} = \langle\partial f/\partial y\rangle\epsilon_1$ is a dimensionless constant, and $\langle\partial f/\partial y\rangle$ denotes the average over the wave function $\varphi(x,y)$.

Next, we further simplify the single electron Hamiltonian by considering the classical limit of the spatial motion of the electron. In the case of large longitudinal momentum of the injected electron compared with the wave number of the TNMM, i.e. $k_z >> q$, the longitudinal motion of the injected electron is hardly affected by the back action of the vibration. Thus, we neglect the recoil effect and used the approximate Galilean transformation $z \to z_0 + vt$, where $v = \hbar k_z/m$ is the velocity of the coherent injected electron. With this approximation, the Hamiltonian can be written as:

$$H = \frac{1}{2}\hbar\omega_z\sigma_z + \hbar\omega_1 a^\dagger a + g_v\cos(\omega_v t + \phi)\left(a^\dagger + a\right)\sigma_x, \tag{2}$$

where $\hbar\omega_z$ is the Zeeman energy of the electron spin. We have ignored the constant term of kinetic energy $\hbar^2k_z^2/2m$. Here, we define the velocity dependent coupling constant $g_v = \alpha\mathcal{E}mv/\hbar$, the frequency $\omega_v = qv$, and the phase $\phi = qz_0$.

There are three different characteristic frequencies in this system: the electron Zeeman frequency ω_z, the NAMR vibration frequency ω_1, and the the spatial motion induced frequency ω_v. In the interaction picture, the Hamiltonian will involve coupling terms with different frequencies. In the spirit of rotating wave approximation, only the terms with the lowest frequency are retained. Therefore, we can adjust the injecting velocity and/or the Zeeman energy of the electrons, so that different types of interaction can be obtained[18]. For example, if $\omega_v \approx \omega_z - \omega_1$, only two terms are retained, and we get a JC-type Hamiltonian (see Fig.2(b)), $H_{\rm JC} = g_v a^\dagger\sigma_-\exp[i(\omega_v t + \phi)]/2 + H.c.$.

Having obtained the actively controllable single electron Hamiltonian, we will study the interactions between the coherently injected ensembles of electron spins and the TNMM of the NAMR. Let N spin polarized electrons with velocity v be injected into the NAMR, and meanwhile, N spin polarized electrons in the opposite direction with the velocity $-v$. In principle, this model can be realized by joining the NAMR with two spin polarized electronic source, for example, two ferromagnetic leads(Fig.2(a)). Then the Hamiltonian for this system is

$$\begin{aligned} H_{\rm sys} &= \hbar\omega_1 a^\dagger a + \frac{1}{2}\hbar\omega_z(J_z + Q_z) \\ &\quad + \tilde{g}_v(t)a^\dagger(J_- - Q_-) + H.c., \end{aligned} \tag{3}$$

where we define the collective spin operators[19]

$$\begin{aligned} J_z &= \sum_{i=1}^N \sigma_z^{(i)}, J_\pm = \sum_{i=1}^N e^{\mp i\phi_i}\sigma_\pm^{(i)}, \\ Q_z &= \sum_{i=1}^N \tau_z^{(i)},\ Q_\pm = \sum_{i=1}^N e^{\mp i\phi_i}\tau_\pm^{(i)}. \end{aligned} \tag{4}$$

Here, the Pauli matrices $\sigma_{z,\pm}^{(i)}$ and $\tau_{z,\pm}^{(i)}$ denote the electron spins with the velocity v and $-v$ respectively, and $\tilde{g}_v(t) = g_v\exp(i\omega_v t)/2$ is the time dependent coupling strength.

Through the Holstein-Primakoff transformation $J_z = -N + 2b^\dagger b$, $J_- = \sqrt{N}b$ and $Q_z = -N + 2c^\dagger c$, $Q_- = \sqrt{N}c$, the collective excitation of the ensembles of electron spins can be characterized by the operators $b(c)$ and $b^\dagger(c^\dagger)$, which satisfy the boson commutative relation in the large N-low excitation limit. Then the interaction between the collective spin excitation and NAMR is modeled by an interacting two-mode boson system. In the

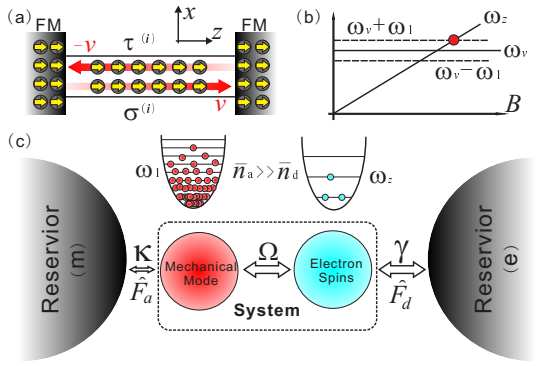

FIG. 2: (a) Polarized electron spins are injected with velocities v and $-v$ into the NAMR from two ferromagnetic (FM) leads. (b) Controllable coupling: the collective excitation energy ω_z of electron spins is linearly dependent on the external magnetic field B, and the frequency ω_v is determined by the electron velocity. The red point denotes the resonant condition $\omega_v = \omega_z - \omega_1$ are satisfied. (c) Schematic illustration of the cooling mechanism. The system contains two interacting boson modes: the fundamental TNMM (red, left) with frequency ω_1 and the collective spin excitation mode (blue, right) with frequency ω_z.

interaction picture,

$$H_I = \hbar\Omega(a^\dagger d e^{-i\Delta t} + H.c.), \tag{5}$$

where $d = (b-c)/\sqrt{2}$ is the boson operator associated with the collective spin excitations, $\hbar\Omega = \sqrt{N/2}g_v$ is the Rabi frequency, and $\Delta = \omega_z - \omega_1 - \omega_v$ is the detuning.

Cooling mechanism.-Next, we study the active cooling mechanism based on the dynamical thermalization of the coupled system of the TNMM and ensembles of electron spins (Fig.2(c)). To describe a practical situation, we assume the TNMM and the ensembles of electron spins to interact with two separated thermal reservoirs. The time evolution of the system operators a and d are governed by the Heisenberg-Langevin equantions[20]

$$\dot{a} = -i\Omega d e^{-i\Delta t} - \frac{\kappa}{2}a + \hat{F}_a(t) \tag{6}$$

$$\dot{d} = -i\Omega a e^{i\Delta t} - \frac{\gamma}{2}d + \hat{F}_d(t) \tag{7}$$

where $\hat{F}_a(t)$ and $\hat{F}_d(t)$ are noise operators due to the reservoirs. We assume the two reservoirs are statistically independent, i.e. $\langle \hat{F}_a^\dagger(t)\hat{F}_d(t')\rangle = 0$. The correlations beteen $\hat{F}_a(t)$ and $\hat{F}_a^\dagger(t)$

$$\begin{aligned}\langle \hat{F}_a^\dagger(t)\hat{F}_a(t')\rangle &= \kappa\bar{n}_a\delta(t-t'),\\ \langle \hat{F}_a(t)\hat{F}_a^\dagger(t')\rangle &= \kappa(\bar{n}_a+1)\delta(t-t')\end{aligned} \tag{8}$$

describe the quantum fluctuations with κ being the damping constant and $\bar{n}_a = [\exp(\hbar\omega_1/k_BT)-1]^{-1}$ being the thermal occupation number of TNMM. The noise operators $\hat{F}_d(t)$ and $\hat{F}_d^\dagger(t)$ follow similar relations with damping constant γ and thermal occupation number $\bar{n}_d = [\exp(\hbar\omega_z/k_BT)-1]^{-1}$.

It follows from Eqs.(6) and (7) that the motions of bilinear quantities $\mathcal{A} = \langle a^\dagger a + a a^\dagger\rangle$, $\mathcal{D} = \langle d^\dagger d + d d^\dagger\rangle$, and $\mathcal{C} = \langle a d^\dagger + d^\dagger a\rangle$ are

$$\dot{\mathcal{A}} = i\Omega\tilde{\mathcal{C}} - i\Omega\tilde{\mathcal{C}}^* - \kappa\mathcal{A} + \kappa\,(2\bar{n}_a + 1)\,, \tag{9}$$

$$\dot{\mathcal{D}} = -i\Omega\tilde{\mathcal{C}} + i\Omega\tilde{\mathcal{C}}^* - \gamma\mathcal{D} + \gamma\,(2\bar{n}_d + 1)\,, \tag{10}$$

$$\dot{\tilde{\mathcal{C}}} = i\Omega\mathcal{A} - i\Omega\mathcal{D} - \frac{1}{2}(\kappa+\gamma)\tilde{\mathcal{C}}, \tag{11}$$

where $\tilde{\mathcal{C}} = \mathcal{C}\exp(i\Delta t)$. The above set of equations determines the time evolution of the occupation numbers (Fig.3). At steady state, the mean occupation number of the TNMM is

$$\langle a^\dagger a\rangle_{\rm ss} = \bar{n}_a - \gamma\,(\bar{n}_a - \bar{n}_d)\,\mathcal{L}(\kappa,\gamma,\Delta), \tag{12}$$

where $\mathcal{L}(\kappa,\gamma,\Delta) = 4\Omega^2(\kappa+\gamma)/\big[\kappa\gamma[(\kappa+\gamma)^2(4\Omega^2/\kappa\gamma + 1) + \Delta^2]\big]$ is a Lorentz line shape function.

The first term in Eq. (12) represents the initial thermal occupation number of the TNMM in equilibrium with the thermal reservoir without injection electron spins. And the second term, which is proportional to the coupling strength Ω^2, is due to the interaction with the spin polarized electrons. It is clear that, if $\bar{n}_a > \bar{n}_d$, the mean occupation number $\langle a^\dagger a\rangle_{\rm ss}$ will be reduced to a value lower than $\bar{n}_a$, or in other words, the TNMM is cooled down. Furthermore, a large cooling efficiency demands the condition $\bar{n}_a >> \bar{n}_d$. It is equivalence to require that (i) the electron Zeeman energy ω_z is much larger than the TNMM frequency ω_1, i.e. $\omega_z >> \omega_1$. This is the first necessary condition for a high cooling efficiency. Under this condition, the mean occupation number at resonance ($\Delta = 0$) can be rewritten as the weighted average of the thermal occupation numbers $\bar{n}_a$ and $\bar{n}_d$:

$$\langle a^\dagger a\rangle_{\rm ss} = (1 - f_1 f_2)\,\bar{n}_a + f_1 f_2 \bar{n}_d \tag{13}$$

with the two ratios $f_1 = 4\Omega^2/(\kappa\gamma + 4\Omega^2)$ and $f_2 = \gamma/(\gamma+\kappa)$. Thus, we find that the mean occupation number $\langle a^\dagger a\rangle_{\rm ss}$ has a neglectably small lower bound $\bar{n}_d$, and a significant reduction of $\langle a^\dagger a\rangle_{\rm ss}$ with respect to the initial thermal equilibrium value $\bar{n}_a$ needs the following two additional requirements: (ii) the coupling between the two modes a and d are strong enough compared with the dampings, i.e. $4\Omega^2 >> \kappa\gamma$, and (iii) the decay rate of the collective electron spin excitations is much larger than the one of the TNMM, i.e. $\gamma >> \kappa$.

To explain the physical mechanism beneath the above protocol, we suppose that the two boson modes are initially in equilibrium with their reservoirs of the same temperature T. Under the condition (i), the initial occupation numbers $\bar{n}_a >> \bar{n}_d$ (see Fig. 2(c)). The condition (ii) ensures the efficient population transfer between the two boson modes. For $t > 0$, the occupation number of the TNMM (collective excitation of electron spins) will

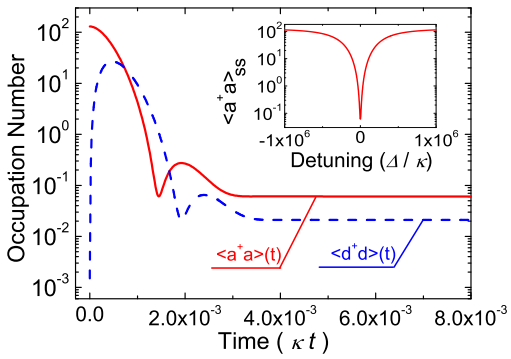

FIG. 3: Time evolution of the occupation numbers $\langle a^\dagger a\rangle(t)$ and $\langle d^\dagger d\rangle(t)$. The occupation numbers of the two boson modes reach steady state values when $t \to +\infty$. Inset: the steady state occupation number $\langle a^\dagger a\rangle_{\rm ss}$ as a function of the detuning Δ. Parameters used in the calculation are described in text.

decrease (increase) due to the coupling (Fig. 3). Simultaneously, the reservoirs tend to maintain the thermal occupation numbers $\bar{n}_a$ and $\bar{n}_d$. In other words, the occupation number of the TNMM (collective excitation electron spins) will gain from (decay to) the reservoir with the rate κ (γ). The condition (iii) defines two different time scales of the thermalization processes. The much faster decay rate γ than κ guarantees that the net effect of the dynamical thermalization is reduction of the occupation number.

Here, we emphasize the importance of the time dependence of coupling $\tilde{g}_v(t)$(see Eq. 3) induced by the spatial motion of spins. Generally speaking, the frequency mismatch of two interacting boson modes will prohibit any efficient couplings. In other words, the requirement (i) would block off the occupation transfer between the two boson modes if their coupling were time independent. Fortunately, in our model, the spatial motion provides a third frequency ω_v to compensate the large frequency difference between ω_1 and ω_z. Thus, the generic resonant condition $\omega_z = \omega_1$ is modified to $\omega_v = \omega_z - \omega_1$. In this way, the spatial motion ensures an efficient coupling under the frequency mismatch condition (i).

Finally, we discuss the experimental feasibility of our cooling mechanism. For a typical NAMR with $l = 2\mu m$, $d = 100\,\text{nm}$, and $h = 80\,\text{nm}$, and the TNMM with frequency $\omega_1 = 300\,\text{MHz}$, the dimensionless constant $\mathcal{E} \approx 10^{-8}$ according to its definitions. With the parameters of GaAs material, mass density $\rho = 5.3g/\,\text{cm}^3$, effective electron mass $m = 0.067m_e$, the spin-orbit interaction constant $\alpha/\hbar = 4\times10^5\,\text{m/s}$, and the velocity of injected electrons $v_F = 1.6\times10^5\,\text{m/s}$, which corresponds to Fermi energy $E_F = 5\,\text{meV}$, we estimate the coupling strength $\Omega = 7\,\text{MHz}$ for $N = 400$. The decay rate γ for the collective excitation of electron spins is determined by the electron spin coherence time $T_1 \sim 50\,\text{ns}$, which implies $\gamma \sim 1/T_1 = 20\,\text{MHz}$. For the quality factor $Q = 10^5$ of the NAMR , and thus $\kappa = \omega_1/Q = 3\,\text{kHz}$, we find the occupation number could reduced from $\bar{n}_a = 130$ without coupling down to $\langle a^\dagger a\rangle_{\rm ss} = 0.06 << 1$ at a sub-Kelvin temperature $T = 300\,\text{mK}$.

Conclusion.-We investigate a spin-TNMM coupling model for cooling the TNMM through dynamical thermalization. We found that the strain induced spin-orbit interactions for conduction band electrons in semiconductor materials result in a microscopic coherent coupling between the electron spins and the quantized TNMM. With this discovery, we present an experimentally accessible cooling method for the TNMM by injecting ensembles of electron spins. The TNMM can be sufficiently cooled down to be capable of exhibiting various quantum phenomena.

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